Dynamic Interaction Factors for Floating Pile Groups

By George Gazetas,[1] Ke Fan,[2] Amir Kaynia,[3] and Eduardo Kausel[4]

Abstract: A comprehensive set of dimensionless graphs of complex-valued dynamic interaction factors versus frequency is presented for vertical, horizontal, and rocking harmonic excitation at the head of each pile. These readily applicable graphs have been developed with a rigorous analytical-numerical formulation for two idealized soil profiles (a homogeneous half-space and a half-space with modulus proportional to depth) and three pile separation distances (3, 5, and 10 pile-diameters). A wide range of values has been parametrically assigned to pile slenderness and pile-to-soil stiffness ratios. The results are discussed at length to gain valuable insight into the nature of dynamic pile-soil-pile interaction. Geotechnical and earthquake engineers can use the presented graphs exactly as they use the classical interaction factors for static deformation analysis of pile groups.

Introduction

Under static working loads, the displacements of a pile increase if this pile is located within the deformation field of a neighboring pile. For a pile group, this leads to an interaction between individual piles, the consequences of which are: (1) The overall stiffness of the group is smaller than the sum of the individual-pile stiffnesses; and (2) the sharing among individual piles of the load applied at the pile cap is generally uneven, with the corner piles loaded the most and center piles loaded the least.

In current geotechnical practice, when the displacement of a pile group is of interest, such pile-soil-pile interaction effects are often assessed through the use of interaction factors, by superimposing the effects of two piles at a time (Poulos 1971). An interaction factor α is defined as the fractional increase in deformation (i.e., deflection or rotation) at the head of a pile due to the presence of a similarly loaded adjacent pile. Thus, if the stiffness of a single (solitary) pile under a given type of loading is $K^{(1)}$, then a load P will produce a deformation $u = P/K^{(1)}$. If two identical piles are each subjected to a load P, then each one will deform by an amount u given by

$$u = \frac{P}{K^{(1)}}(1 + \alpha) \quad \text{(1)}$$

The value of α depends, of course, on the type of loading (axial or lateral), the spacing of the two piles, and the soil and pile material and geometric properties.

The popularity of this superposition method stems from the availability (in published form) of fairly complete sets of static interaction factors, developed by recourse to integral equation and finite element formulations

---

[1]Prof., Dept. Civ. Engrg., State Univ. of New York, Buffalo, NY 14260.
[2]Grad. Student, Dept. Civ. Engrg., State Univ. of NY, Buffalo, NY 14260.
[3]Assoc. Prof., Dept. Civ. Engrg., Isfahan Univ. of Tech., Isfahan, Iran.
[4]Prof., Dept. Civ. Engrg., Massachusetts Inst. of Tech., Cambridge, MA 02139.

Note. Discussion open until March 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on September 6, 1990. This paper is part of the *Journal of Geotechnical Engineering*, Vol. 117, No. 10, October, 1991. ©ASCE, ISSN 0733-9410/91/0010-1531/$1.00 + $.15 per page. Paper No. 26250.

(Poulos 1968, 1971; Butterfield and Banerjee 1971; Poulos and Davis 1980) and to simple physically sound approximations (Randolph and Wroth 1979).

Unfortunately, the static interaction factors are not applicable to the dynamic analysis of pile groups, except perhaps at very low frequencies of oscillation. Indeed, dynamic studies of pile groups (Wolf and Von Arx 1978; Waas and Hartmann 1981; Kaynia and Kausel 1982; Sheta and Novak 1982; Nogami 1983; Kagawa 1983; Tyson and Kausel 1983; Roesset 1984; Dobry and Gazetas 1988) have demonstrated that the dynamic response of pile groups may differ substantially from their static response, in that the dynamic group efficiency exhibits a strong sensitivity to frequency and may attain values well above unity. (Dynamic group efficiency is defined as the ratio of total group dynamic stiffness to the sum of the individual pile stiffnesses.) Nevertheless, Kaynia and Kausel (1982) have shown that even for dynamic loads, Poulos' superposition procedure remains an excellent engineering approximation, provided of course that dynamic interaction factors are used for each frequency of interest. Today, despite the significant progress in understanding dynamic pile group behavior, only a very limited number of dynamic interaction factors have been published in a form readily accessible to practicing geotechnical and earthquake engineers. In response to this apparent need, a comprehensive set of dimensionless graphs of dynamic interaction factors have been developed and are presented in this paper as functions of frequency for a practically sufficient range of key material and geometric parameters. These graphs may be readily used in practice to obtain estimates of the dynamic response of floating pile groups, for soil deposits that could be modeled either as a homogeneous deep stratum or a deposit with stiffness proportional to depth.

Definitions and Method of Solution

Fig. 1 shows the system studied: two identical vertical free-head piles, floating in a half-space with Young's modulus either constant E_s, or proportional to depth, $E_s(z) = E_s(L)z/L$. The piles, of diameter d and length L, are considered to be linear-elastic beams with constant Young's modulus E_p and mass density ρ_p. The soil is assumed to be a linear-hysteretic continuum with constant Poisson's ratio ν_s, constant material density ρ_s, and constant hysteretic damping β_s; unless otherwise noted, the following typical values were assigned to these three parameters: $\nu_s = 0.40$, $\beta_s = 0.05$, and $\rho_s = 0.70\rho_p$. However, the values of $\rho_s = 1.2\rho_p$, typical of hollow cylindrical piles, and $\nu_s = 0.48$, typical for saturated clays, are also given consideration. Finally, S = the axis-to-axis spacing of the piles; and θ = their angle of "departure," i.e., the angle between the line joining the pile centers and the direction of loading (Fig. 1).

For two such piles, a frequency-dependent dynamic interaction factor, $\alpha = \alpha(\omega)$, is defined as

$$\alpha = \frac{\text{dynamic displacement of pile 2 caused by pile 1}}{\text{static displacement of solitary pile 1 due to its own load}} \quad \ldots\ldots\ldots (2)$$

in which displacement means translation or rotation. Five different kinds of dynamic interaction factors are provided in this paper, depending on the loading at the pile head and the type of deformation.

1. α_v = interaction factor for vertical deflection under vertical loading.

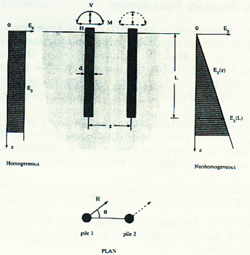

FIG. 1. Sketch of System Studied

2. α_{uH} = interaction factor for horizontal deflection of free-headed piles under horizontal force loading.
3. $\alpha_{\phi M}$ = interaction factor for rotation of free-headed piles under moment loading.
4. $\alpha_{uM} = \alpha_{\phi H}$ = interaction factors for horizontal deflection due to moment or for rotation due to horizontal force of free-headed piles.

The results of the presented graphs of dynamic interaction factors were obtained with a rigorous analytical-numerical formulation developed by Kaynia and Kausel (1982) for the dynamic analysis of pile groups in a layered half-space. In addition, the simplified analytical method of Dobry and Gazetas (1988) and Gazetas and Makris (1991) was used in selecting suitable dimensionless problem parameters, in assessing the effects of some of these parameters, and in explaining certain trends observed in the rigorous results.

The Kaynia and Kausel (1982) formulation is in essence a boundary-integral type method in which the Green's functions, defining the displacement fields due to uniform barrel and disk loads associated with pile-soil interface tractions, are computed by solving the wave equations through Fourier and Hankel transformations (Kausel 1981). These functions yield the dynamic soil flexibility matrix, which is combined with the analytically derived pile flexibility matrix, while compatibility of deformations at the pile-soil interface is enforced. (No gap is allowed between pile and soil, an assumption that might not be appropriate in the case of strong excitation and certain types of soils.)

Dimensionless Problem Parameters

The two key dimensionless parameters that have been shown in the literature to largely control the value of all interaction factors are: (1) The frequency factor $a_0 = (\omega d/V_s^*)$, where V_s^* = a characteristic value of the soil S-wave velocity profile (in this paper V_s^* is taken equal to V_s for the homogeneous and to $V_s(L)$ for the inhomogeneous profiles); and (2) the pile spacing-to-diameter ratio (hereafter called simply the spacing ratio) s/d.

The curves of the interaction factors as functions of the frequency factor, i.e., $\alpha = \alpha(a_0)$, exhibit peaks and troughs occurring at different locations for different values of the spacing ratio s/d [e.g., Kaynia and Kausel (1982)]. The picture clears up significantly when an alternative frequency parameter

$$b_0 = a_0 \frac{s}{d} = \frac{\omega s}{V_s^*} \quad (3)$$

is used in place of a_0. Indeed, according to the aforementioned simplified method of Dobry and Gazetas (1988) as a first approximation, interaction factors take the form

$$\alpha \approx \frac{1}{\sqrt{2}} \left(\frac{s}{d}\right)^{-1/2} \exp\left(\frac{-\beta_s \omega s}{V}\right) \cdot \exp\left(\frac{-i\omega s}{V}\right) \quad (4)$$

where $i = \sqrt{-1}$; and V is equal to, or a multiple of, V_s^*, depending on the mode of deformation and the type of soil profile. The first two terms in (4) constitute the amplitude, while the last term controls the undulations with frequency, of the interaction factor. It is evident that the amplitude $|\alpha|$ decreases with increasing spacing ratio s/d and with increasing soil hysteretic damping β_s and frequency parameter b_0. On the other hand, the fluctuations of α with frequency depend solely on the frequency parameter b_0. Hence, the graphs of this paper are in the form $\alpha = \alpha(b_0)$.

The third most important problem parameter appears to be the so-called angle of "departure" θ for the lateral interaction factors. Following Poulos (1971) and Kausel and Kaynia (1982), results are presented here only for $\theta = 0°$ and $\theta = 90°$. For any other angle, the interaction factors can be obtained with sufficient accuracy from the following relationship:

$$\alpha(\theta°) \approx \alpha(0°)\cos^2\theta + \alpha(90°)\sin^2\theta \quad (5)$$

The vertical interaction factor is, of course, independent of θ due to symmetry.

The two other parameters that have been found to have an influence (although rather secondary, for practical purposes) on the interaction factors are: (1) The ratio of the "effective" pile modulus to soil Young's modulus E_p/E_s or $E_p/E_s(L)$ for the homogeneous or inhomogeneous profiles of Fig. 1; and (2) the pile slenderness ratio L/d.

Note that the presented graphs, although derived for circular piles of solid cross section with diameter d, can also be used for pipe piles and concrete-filled steel pipe piles (Gazetas and Dobry 1984). To this end, an appropriate "effective" modulus E_p is chosen such that

for axial deformations:

$$E_p = \frac{(EA)_p}{\pi r_0^2} \quad (6a)$$

for lateral deformations:

$$E_p = \frac{(EI)_p}{\frac{\pi r_0^4}{4}} \quad \text{(6b)}$$

where $(EA)_p$ and $(EI)_p$ = the axial and bending rigidities of the actual pile section. [For example, for a steel pipe pile: $(EA)_p = E_{steel}(\pi r_0^2 - \pi r_i^2)$ and $(EI)_p = E_{steel}(\pi r_0^4 - \pi r_i^4)/4$; $r_0 = d/2$ = external radius, and r_i = internal radius of the pipe.]

Graphs

Figs. 2–18 show the real and imaginary parts of the complex-valued dynamic interaction factors as functions of frequency, in the following dimensionless parametric form:

$$\alpha = \text{Real}(\alpha) + i \,.\, \text{Imaginary}(\alpha) \quad \text{(7a)}$$

$$\alpha = \alpha\left(b_0; \frac{s}{d}, \theta°, \frac{E_p}{E_s} \text{ or } \frac{E_p}{E_s(L)}, \frac{L}{d}\right) \quad \text{(7b)}$$

Organization of the graphs and the considered ranges of problem parameters are presented in Table 1. Note that b_0 is given values up to five, which would be sufficiently high for most applications, even when pile spacing equals 10.

The following characteristic trends are worthy of note in the graphs of Figs. 2–18.

Dynamic interaction factors are quite different from the respective static interaction factors, to which they converge only at zero frequency. It is apparent that use of static interaction factors in estimating the dynamic response of pile groups must be avoided as it would, in general, worsen rather than improve the prediction, i.e., one would be better off ignoring interaction altogether.

While static interaction factors are invariably positive numbers smaller than unity, the dynamic factors are complex with real and imaginary parts, Re(α) and Im(α), that fluctuate with frequency, achieving positive and negative values. As the aforementioned simplified method (4) has anticipated, this frequency dependence of α is indeed almost entirely controlled by the frequency parameter b_0. Of particular significance for the response of pile groups are the negative values of Re(α). Such values arise whenever waves originating from pile 1 with a certain phase arrive at a neighboring pile 2 in an exactly opposite phase, thereby inducing displacements v_{12} which are opposite to the displacements v_{22} due to this pile's own load. As a consequence, in a pile group loaded, e.g., through a rigid cap, a larger force must be applied to pile 2 to enforce a certain displacement amplitude; thus, the dynamic stiffness of the group increases and may achieve values well above the sum of the individual stiffness of each pile ("efficiency" greater than unity, in established geotechnical terminology).

The soil Poisson's ratio ν_s, and the soil-to-pile mass density ratio ρ_s/ρ_p have no discernible effect on dynamic interaction factors. On the other hand, the effect of soil hysteretic damping β_s can be captured with the simplified expression given in (4)

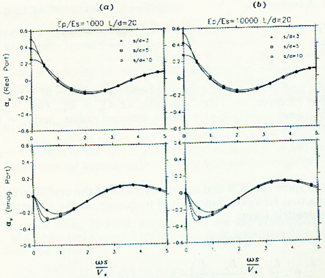

FIG. 2. Vertical Interaction Factor for: (a) Relatively Compressible Piles (E_p/E_s = 1,000); (b) Relatively Rigid Piles (E_p/E_s = 10,000) in Homogeneous Soil (L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4)

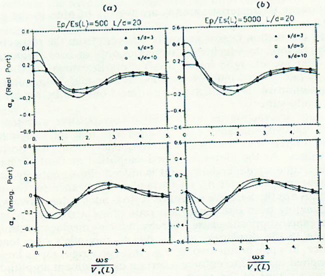

FIG. 3. Vertical Interaction Factor for (a) Relatively Compressible Piles ($E_p/E_s(L)$ = 500); (b) Relatively Rigid Piles ($E_p/E_s(L)$ = 5,000) in Nonhomogeneous Soil (L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4)

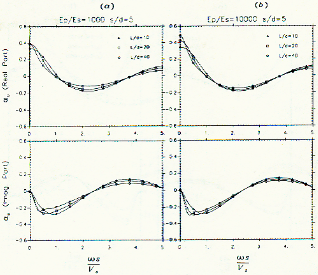

FIG. 4. Effect of L/d on Vertical Interaction Factor for: (a) Relatively Compressible Piles (E_p/E_s = 1,000); (b) Relatively Rigid Piles (E_p/E_s = 10,000) in Homogeneous Soil (s/d = 5, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4)

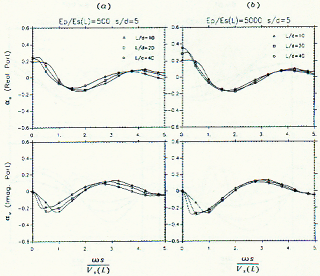

FIG. 5. Effect of L/d on Vertical Interaction Factor for: (a) Relatively Compressible Piles ($E_p/E_s(L)$ = 500); (b) Relatively Rigid Piles ($E_p/E_s(L)$ = 5,000) in Nonhomogeneous Soil (s/d = 5, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4)

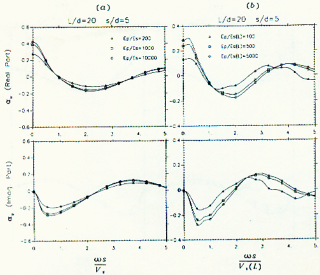

FIG. 6. Effect of E_p/E_s on Vertical Interaction Factor for Piles in: (a) Homogeneous Soil; (b) Nonhomogeneous Soil (L/d = 20, s/d = 5, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4)

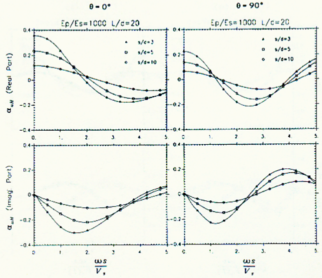

FIG. 7. Horizontal Interaction Factor for Relatively Compressible Piles in Homogeneous Soil (E_p/E_s = 1,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

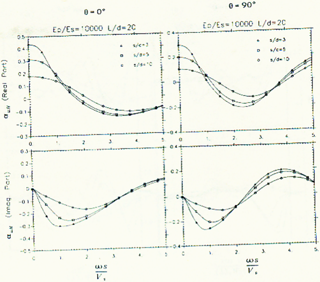

FIG. 8. Horizontal Interaction Factor for Relatively Rigid Piles in Homogeneous Soil (E_p/E_s = 10,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

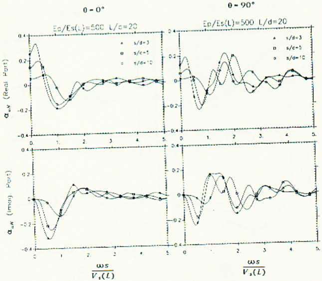

FIG. 9. Horizontal Interaction Factor for Relatively Compressible Piles in Nonhomogeneous Soil ($E_p/E_s(L)$ = 500, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

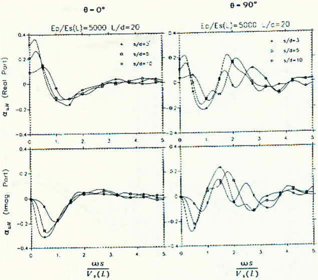

FIG. 10. Horizontal Interaction Factor for Relatively Rigid Piles in Nonhomogeneous Soil ($E_p/E_s(L)$ = 5,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

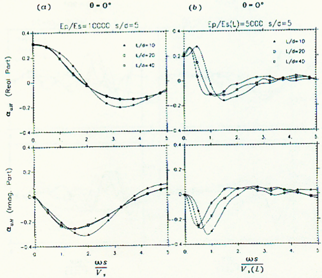

FIG. 11. Effect of L/d on Horizontal Interaction Factor for Relatively Rigid Piles in: (a) Homogeneous Soil (E_p/E_s = 10,000); (b) Nonhomogeneous Soil ($E_p/E_s(L)$ = 5,000) (s/d = 5, ρ_s/ρ_p = 0.7, β = 0.05, ν_s = 0.4, and θ = 0°)

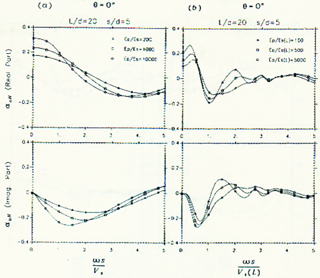

FIG. 12. Effect of E_p/E_s on Horizontal Interaction Factor for Piles in: (a) Homogeneous Soil; (b) Nonhomogeneous Soil (L/d = 20, s/d = 5, ρ_s/ρ_p = 0.7, β = 0.05, ν_s = 0.4, and θ = 0°)

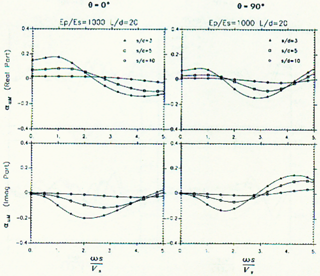

FIG. 13. Coupling Interaction Factor for Relatively Compressible Piles in Homogeneous Soil (E_p/E_s = 1,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

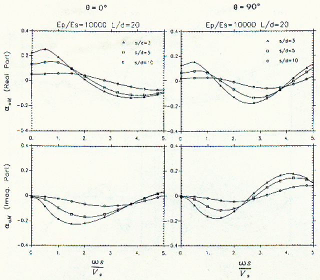

FIG. 14. Coupling Interaction Factor for Relatively Rigid Piles in Homogeneous Soil (E_p/E_s = 10,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

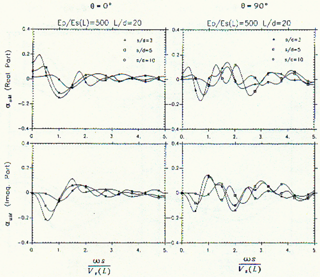

FIG. 15. Coupling Interaction Factor for Relatively Compressible Piles in Non-homogeneous Soil ($E_p/E_s(L)$ = 500, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

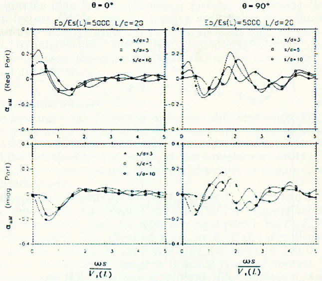

FIG. 16. Coupling Interaction Factor for Relatively Rigid Piles in Nonhomogeneous Soil ($E_p/E_s(L)$ = 5,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

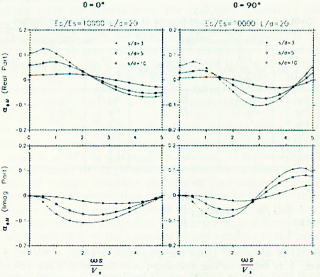

FIG. 17. Rotational Interaction Factor for Relatively Rigid Piles in Homogeneous Soil (E_p/E_s = 10,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

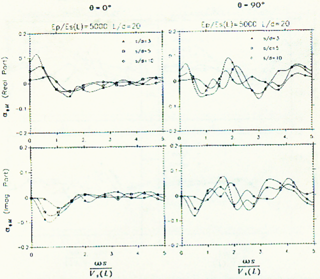

FIG. 18. Rotational Interaction Factor for Rigid Piles in Nonhomogeneous Soil [$E_p/E_s(L)$ = 5,000, L/d = 20, ρ_s/ρ_p = 0.7, β = 0.05, and ν_s = 0.4): (a) θ = 0°; (b) θ = 90°

TABLE 1. Organization of Graphs of Dynamic Interaction Factors

Interaction factors (1)	Homogeneous Profile: s/d 3, 5, 10 (2)	Homogeneous Profile: L/d 10, 20, 40 (3)	Homogeneous Profile: E_p/E_s 200, 1,000, 10,000 (4)	Nonhomogeneous Profile: s/d 3, 5, 10 (5)	Nonhomogeneous Profile: L/d 10, 20, 40 (6)	Nonhomogeneous Profile: $E_p/E_s(L)$ 100, 500, 5,000 (7)
α_v	Fig. 2	Fig. 4	Fig. 6	Fig. 3	Fig. 5	Fig. 6
α_{uH}	Fig. 7	Fig. 11	Fig. 12	Fig. 9	Fig. 11	Fig. 12
	Fig. 8	—	—	Fig. 10	—	—
$\alpha_{uM} = \alpha_{\phi H}$	Fig. 13	—	—	Fig. 15	—	—
	Fig. 14	—	—	Fig. 16	—	—
$\alpha_{\phi M}$	Fig. 17	—	—	Fig. 18	—	—

$$\frac{\alpha(\beta_s)}{\alpha(0.05)} \approx \exp\left[\frac{-(\beta_s - 0.05)\omega s}{V}\right] \quad (8)$$

where $\alpha(0.05)$ is obtained from the provided graphs, while as a first approximation, V is taken as the average (over depth) S wave velocity $\overline{V}_s$, or the average Lysmer's analog velocity, $\overline{V}_{La} = [3.4/\pi(1 - \nu_s)]\overline{V}_s$. Specifically, $V = \overline{V}_s$ for the vertical interaction factor, and for the lateral interaction factors when $\theta = 90°$, while $V = \overline{V}_{La}$ for the lateral factors when $\theta = 0°$ [see Dobry and Gazetas (1988)]. Evidently, the effect of damping may become significant only at high values of the frequency parameter b_0, i.e., at high frequencies and/or large spacing ratios.

Under static and low-frequency conditions, vertical interaction factors are generally greater than lateral interaction factors, and among the latter, interaction factors due to moment loading are smaller than those due to horizontal-force loading, while for rotation they are smaller than for displacement. At intermediate and higher frequencies, however, due to the observed undulations, this general picture may not be true, except perhaps in a general average sense.

Vertical Interaction Factors

The pile spacing ratio s/d affects α_v in two different ways: (1) By controlling its static and low-frequency amplitude; and (2) by influencing the frequencies at which peaks and troughs occur. By contrast, for a homogeneous soil the amplitudes of those peaks and troughs are hardly influenced by s/d; with inhomogeneous soil, the effect of s/d becomes appreciable only for s/d exceeding five.

The stiffness and slenderness ratios, E_p/E_s [or $E_p/E_s(L)$] and L/d, are of secondary importance. Specifically, the effect of L/d is appreciable at zero and very low frequencies only for relatively rigid piles [i.e., $E_p/E_s \approx 10{,}000$ or $E_p/E_s(L) \approx 5{,}000$]. The effect of E_p/E_s or $E_p/E_s(L)$ is noticeable only when these ratios attain very low values (of the order of 200 or 100, respectively), i.e., in the lowest range of possible practical interest.

Under static and low-frequency conditions, the smallest α_v values are associated with the inhomogeneous deposit. At moderate and high frequencies, however, inhomogeneity affects mainly the location and shape of the peaks and valleys of α_v, rather than their amplitude.

Lateral Interaction Factors

The spacing ratio s/d affects both the real and imaginary part of the amplitudes of lateral interaction factors over the whole frequency range studied. (This contrasts with the behavior of α_v, the amplitude of which is affected by s/d only at zero and very low frequencies.) One of the consequences is that for $s/d = 10$, α_{uH}, the largest of the lateral interaction factors, attains at all frequencies very small values (e.g., less than 0.10 for $E_p/E_s = 1{,}000$), whereas the corresponding vertical factor α_v for frequencies b_0 exceeding one, achieves for $s/d = 10$ essentially identical (relatively high) values with those for $s/d = 3$.

The effect of the angle of "departure" θ is twofold. First, the static and low-frequency amplitudes of all lateral interaction factors decrease as θ increases, e.g., $\alpha_{uH}(90°) \approx 0.60\alpha_{uH}(0°)$. Second, at intermediate and high frequencies, increasing the angle θ does not produce a decreasing amplitude of the peaks and valleys; however, the rate of fluctuation (of both real and imaginary parts) of the interaction factor is faster for $\theta = 90°$ than for $\theta = 0°$. To provide a physical explanation of this effect, recall that according to (4) of the aforementioned simplified model, the fluctuations of α are controlled by $\omega s/V$, where $V =$ the (average) wave velocity of the predominant waves. For $\theta = 90°$, one pile sends to the other mainly S waves and, thus, $V = V_s$ in a homogeneous deposit. For $\theta = 0°$, the two piles interact through compression-extension, rather than shear, waves; such waves propagate at an apparent phase velocity $V \approx V_{La}$ [defined as Lysmer's analog velocity by Gazetas and Dobry (1984)]. In this case, $V_{La} = [3.4/\pi(1 - \nu)] \cdot V_s \approx 1.80V_s$, leading to rates of fluctuation $\omega s/V_s$ (for $\theta = 90°$) $= 1.80$ times $\omega s/V_{La}$ (for $\theta = 0°$), in accord with the observed faster rate of fluctuations for $\theta = 90°$.

It is well understood that the slenderness ratio L/d plays no role in the lateral response of flexible piles, i.e., piles of which the length L exceeds a "critical" or "active" length l_c given by the following conservative expression (Gazetas 1991):

$$l_c \approx 2d\left(\frac{E_p}{E_s}\right)^{0.25} \quad \text{(9)}$$

applicable for homogeneous soils. The part of the pile located below l_c from the top remains practically idle at all frequencies and, therefore, pile response and pile-to-pile interaction are governed by l_c and not L. Most real-life piles, as well as the piles in our parametric study, are indeed "flexible." For example, with $E_p/E_s = 1{,}000$, the "active" length becomes $l_c \approx 2d(1{,}000)^{0.25} \approx 11.2d$; thus, only the shortest of the considered piles, with $L = 10d$, falls just below the limit for completely "flexible" behavior.

The obtained results (only some of which are plotted in Fig. 11) confirm that L/d has no effect on lateral interaction factors for all but the decisively "rigid" piles. Only the shortest and stiffest of the considered piles $L/d = 10$ and $(E_p/E_s = 10{,}000)$ belong in that category, and thereby their α_{uH} show some differences from the single α_{uH} function for all the other piles.

This last conclusion is also, in general, valid for the nonhomogeneous soil profile. The "active" length in this case is given by the following conservative expression (Gazetas 1991):

$$l_c \approx 2d\left(\frac{E_p}{E_s}\right)^{0.20} \quad \text{(10a)}$$

where

$$E_s = E_s(d) = E_s(L)\cdot\frac{d}{L} \quad \text{(10b)}$$

For instance, the typical pile with $L/d = 20$ and $E_p/E_s(L) = 500$ has an $l_c \approx 2d(500 \times 20)^{0.20} \approx 12.6d$; thus, $L > l_c$, and this pile is "flexible." Almost "flexible" is the $L/d = 10$ pile, for which $l_c \approx 2d(500 \times 10)^{0.20} \approx 11d$. Again, only the stiffest and shortest of the considered piles $[E_p/E_s(L) = 5{,}000$ and $L/d = 10]$ are decisively "rigid" piles: $l_c \approx 17.5d > 10d$; their interaction factors α_{uH} show some (small) differences in peak amplitudes from those of α_{uH} for the $L = 20d$ and $L = 40d$ piles.

The small frequency shift observed in Fig. 11(b) (as L/d decreases, peaks and valleys move towards larger b_0) is of absolutely no significance, being merely an artifact of plotting versus $b_0 = \omega s/V_s(L)$. Indeed, the reader should appreciate that the exact value of $V_s(L)$ is of no relevance to the interaction of "flexible" piles. If a relevant wave velocity, e.g., that at one diameter depth $V_s(d) = V_s(L)\cdot(d/L)^{1/2}$, were to be used instead, this frequency shift would disappear, since the abscissa would change to

$$\frac{\omega s}{V_s(d)} = b_0\cdot\left(\frac{L}{d}\right)^{1/2} \quad \text{(11)}$$

and thus the interaction curves of the longer piles would move farther to the right, thereby meeting the corresponding curves of the shorter piles.

Increasing the stiffness ratio E_p/E_s or $E_p/E_s(L)$ produces an appreciable increase in all lateral interaction factors under static and low-frequency loading. At higher frequencies, however, the increase is somewhat less significant.

Note that, for static loading, Randolph (1977) has proposed an approximate expression for α_{uH}, which for homogeneous soil takes the form

$$\alpha_{uH} \approx 0.28\left(\frac{s}{d}\right)^{-1}\left(\frac{E_p}{E_s}\right)^{1/7}(1 + \cos^2\theta) \qquad (12)$$

and fits closely the zero-frequency results of this study.

The two rotational interaction factors, α_{uM} and $\alpha_{\phi M}$, attain very small values for all but the closest possible spacing ratios ($s/d \leq 3$). For static loading, Randolph (1977) proposed the following appproximations

$$\alpha_{uM} = \alpha_{\phi H} \approx \alpha_{uH}^2 \qquad (13a)$$

$$\alpha_{\phi M} \approx \alpha_{uH}^3 \qquad (13b)$$

It appears that these relations hold approximately true even for dynamic loading and could be recommended at least in routine practical applications.

Conclusion

Graphs of dynamic interaction factors for vertical and horizontal displacements and rotations of free-head piles embedded in homogeneous and nonhomogeneous half-spaces have been presented. These results should be of practical value in the seismic design of pile foundations and in the seismic analysis of soil-structure interaction. The presented graphs can be readily applied by engineers already familiar with the use of static interaction factors in the design of pile groups.

Appendix. References

Butterfield, R., and Banerjee, P. K. (1971). "The elastic analysis of compressible piles and pile groups." *Geotechnique*, 21(1), 43–60.

Dobry, R., and Gazetas, G. (1988). "Simple method for dynamic stiffness and damping of floating pile groups." *Geotechnique*, 38(4), 557–574.

Gazetas, G. (1991). "Foundation vibrations." *Foundation engineering handbook*, 2nd Ed., Y. Fang, ed., Van Nostrand Reinhold Co., New York, N.Y., 553–593.

Gazetas, G., and Dobry, R. (1984). "Horizontal response of piles in layered soils." *J. Geotech. Engng.*, ASCE, 110(1), 20–40.

Gazetas, G., and Makris, N. (1991). "Dynamic pile-soil-pile interaction. I: Analysis of axial vibration." *Earthquake Engrg. Struct. Dyn.*, 20(2), 115–132.

Kagawa, T. (1983). "Dynamic lateral pile group effects." *J. Geotech. Engng.*, ASCE, 109, 1267–1285.

Kaynia, A. M., and Kausel, E. (1982). "Dynamic stiffnesses and seismic response of pile groups." *Research Report*, Dept. of Civil Engineering, MIT, Cambridge, Mass.

Kausel, E. (1981). "An explicit solution for the Green functions for dynamic loads in layered media." *Research Report*, Dept. of Civil Engrg., MIT, Cambridge, Mass.

Nogami, T. (1983). "Dynamic group effect in axial responses of grouped piles." *J. Geotech. Engng. Div.*, ASCE, 109, 225–243.

Poulos, H. G. (1968). "Analysis of the settlement of pile groups." *Geotechnique*, 18(4), 449–471.

Poulos, H. G. (1971). "Behavior of laterally loaded piles. II: Pile groups." *J. Soil Mech. Found. Engrg. Div.*, ASCE, 97(5), 773–751.

Poulos, H. G., and Davis, E. H. (1980). *Pile foundation analysis and design.* Wiley, New York, N.Y.

Randolph, M. F. (1977). "A theoretical study of the performance of piles." thesis presented to Cambridge University, at Cambridge, England, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
Randolph, M. F., and Wroth, C. P. (1979). "An analysis of the vertical deformation of pile groups." *Geotechnique*, 29(4), 423–439.
Roesset, J. M. (1984). "Dynamic stiffness of pile groups." *Pile foundations*, ASCE, New York, N.Y., 263–286.
Sheta, M., and Novak, M. (1982). "Vertical vibration of pile groups." *J. Geotech. Engng. Div.*, ASCE, 108, 570–590.
Tyson, T. R., and Kausel, E. (1983). "Dynamic analysis of axisymmetric pile groups." *Research Report*, Dept. of Civil Engrg., MIT, Cambridge, Mass.
Waas, G., and Hartmann, H. G. (1981). "Analysis of pile foundations under dynamic loads." *Conf. of Struct. Mech. in Reactor Tech.*, Paris, France.
Wolf, J. P., and Von Arx, G. A. (1978). "Impedance function of a group of vertical piles." *Soil dynamics and Earthquake Engineering*, ASCE, New York, N.Y., Vol. 2, 1024–1041.